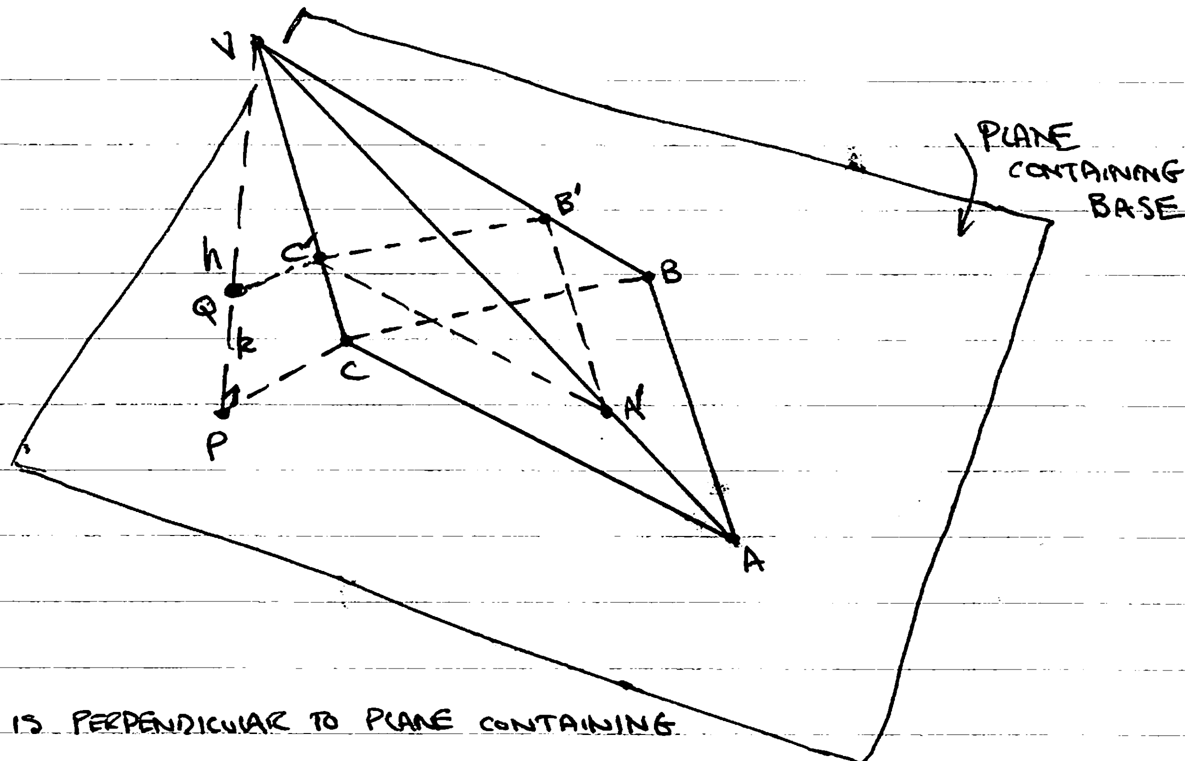


$\overline{VP}$ IS PERPENDICULAR TO PLANE CONTAINING BASE TRIANGLE ΔABC. h = LENGTH OF $\overline{VP}$ = HEIGHT OF PYRAMID.

INTERSECT THE PYRAMID WITH A PLANE PARALLEL TO THE BASE PLANE AT A HEIGHT k, $0 \le k \le h$, ABOVE THE BASE PLANE. THIS PLANE WILL INTERSECT THE ~~BASE~~ PYRAMID IN A TRIANGLE $\Delta A'B'C'$

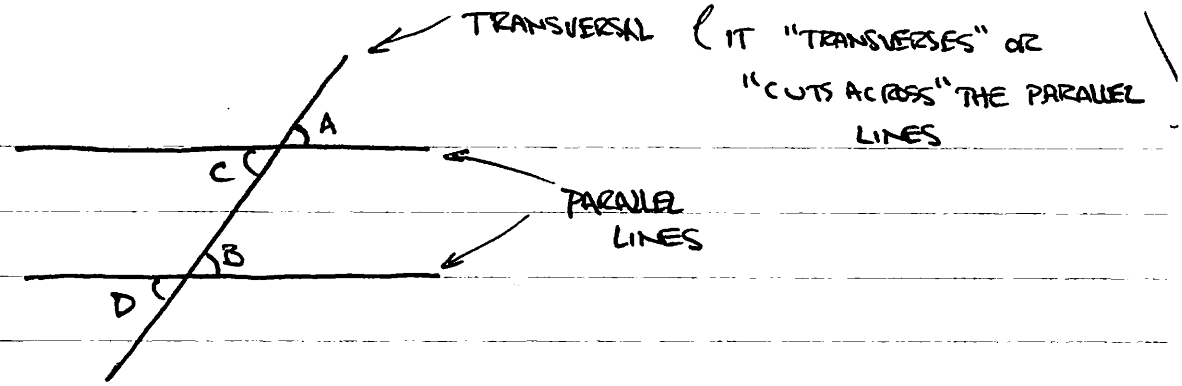
WANT TO SHOW $\Delta ABC \sim \Delta A'B'C'$. WE CAN SHOW TWO TRIANGLES ARE SIMILAR IF WE CAN SHOW EITHER:

EQUIVALENT STATEMENTS

① CORRESPONDING ANGLES OF THE TWO TRIANGLES ARE EQUAL, OR

② RATIOS OF THE CORRESPONDING SIDES OF THE TWO TRIANGLES ARE EQUAL.

TO SHOW ① WE OFTEN USE THE FACT THAT IF PARALLEL LINES ARE INTERSECTED BY A TRANSVERSAL THEN CORRESPONDING ANGLES ARE EQUAL AND ALTERNATE INTERIOR AND ALTERNATE EXTERIOR ANGLES ARE EQUAL (SEE FIGURE)



A AND B ARE CORRESPONDING ANGLES (THEY BEAR THE SAME INTERSECTING RELATIONSHIP TO EACH OF THE PARALLEL LINES)

B AND C ARE ALTERNATE INTERIOR ANGLES

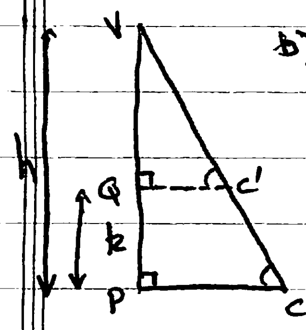
A AND D ARE ALTERNATE EXTERIOR ANGLES

A AND C ARE CALLED "VERTICAL" ANGLES (VERTICAL ANGLES ARE ALSO EQUAL)

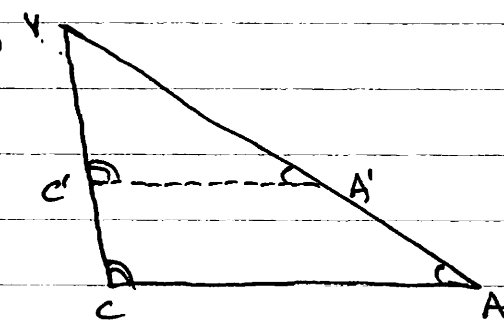
WE WILL SHOW $\Delta ABC \sim \Delta A'B'C'$ USING CONDITION ②.

CONSIDER THE FOUR TRIANGLES (THE THREE SIDES OF THE PYRAMID AND ΔVPC) BELOW:

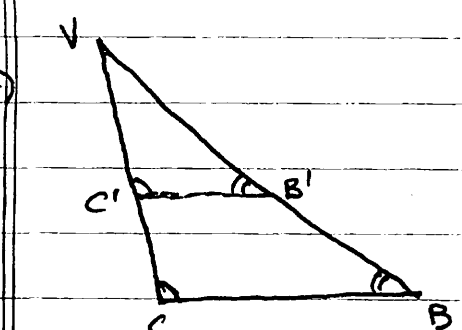
a)



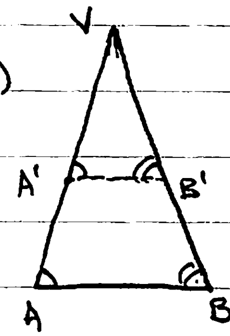
b)



c)



d)



SINCE SEGMENTS $\overline{QC'}$, $\overline{C'A'}$, $\overline{A'B'}$ AND $C'B'$ LIE IN A PLANE PARALLEL TO BASE, WHICH ALSO CONTAINS SEGMENT $\overline{PC}$, WE HAVE:

$\overline{QC'} \parallel \overline{PC}$, $\overline{C'A'} \parallel \overline{CA}$,

$\overline{C'B'} \parallel \overline{CB}$, $\overline{A'B'} \parallel \overline{AB}$

IT FOLLOWS FROM ① THAT

a) $\Delta VQC' \sim \Delta VPC$,

b) $\Delta VC'A' \sim \Delta VCA$,

c) $\Delta VC'B' \sim \Delta VCB$ AND

d) $\Delta VA'B' \sim \Delta VAB$

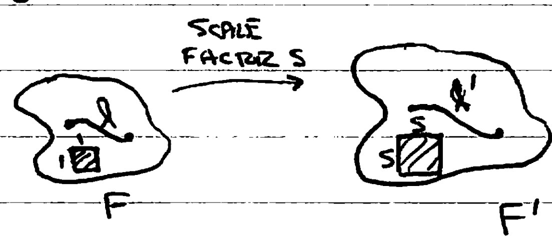
USING STATEMENT ② AND SIMILAR TRIANGLES a), b), c), d) WE HAVE:

$$\frac{VQ}{VP} = \frac{h-k}{h} \overset{(a)}{=} \frac{VC'}{VC} \overset{(b)}{=} \frac{C'A'}{CA} \overset{(b)}{=} \frac{VA'}{VA}$$

BUT ALSO: $\frac{VC'}{VC} \overset{(c)}{=} \frac{C'B'}{CB}$ AND $\frac{VA'}{VA} \overset{(d)}{=} \frac{A'B'}{AB}$

HENCE $\frac{h-k}{h} = \frac{C'A'}{CA} = \frac{C'B'}{CB} = \frac{A'B'}{AB}$. SINCE RATIOS OF CORRESPONDING SIDES IN $\triangle ABC$ AND $\triangle A'B'C'$ ARE ALL EQUAL $\triangle ABC \sim \triangle A'B'C'$.

NEXT, IF TWO FIGURES F AND F' ARE SIMILAR WITH SCALING FACTOR S



$F \sim F'$ WITH SCALE FACTOR $S \longrightarrow \frac{\ell'}{\ell} = S$

THEN THE AREAS SCALE AS S^2 I.E AREA OF $F' = S^2$ (AREA OF F)

OR EQUIVALENTLY:

$$\frac{\text{AREA OF } F'}{\text{AREA OF } F} = S^2 = \left(\frac{\ell'}{\ell}\right)^2$$

WE CONCLUDE, THEREFORE, THAT AREA $\triangle A'B'C' = S^2$ (AREA $\triangle ABC$) WHERE $S = \frac{C'A'}{CA} = \frac{h-k}{h}$.