Math 110 100/100 **Final Exam** Name KEY

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Be sure to quickly look through the exam to get an idea of its length and the type of problems involved. First do the problems with which you have confidence, then attack other less obvious ones. Be neat and show only the work you want me to see. Please circle your final solutions. No credit is given for solutions only.

9@6 EACH 1. Perform the indicated operations and simplify each expression. Combine like terms where possible, reduce any fractions to lowest terms, and express your solution using positive exponents only.

(a) $-2(3x^2 + x - 1) - x(4 - 6x)$

$= -6x^2 - 2x + 2 - 4x + 6x^2 = \boxed{-6x + 2}$

(b) $(4x^3)^2 \cdot (2x)^{-3}$

$= 16x^6 \cdot 2^{-3} x^{-3} = \frac{16x^3}{8} = \boxed{2x^3}$

(c) $\frac{(2wy^{-3})^4}{6y^{-3}} = \frac{16w^4 y^{-12}}{6y^{-3}} = \frac{8}{3} w^4 y^{-9} = \boxed{\frac{8w^4}{3y^9}}$

(d) $(5x + 2)^2 = \boxed{25x^2 + 20x + 4}$

(e) $(a - b)(a^2 + ab + b^2)$

$a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3 = \boxed{a^3 - b^3}$

(f) $\frac{m^2 - 4}{m^2 + 5m + 6} \cdot \frac{2m^2 + 6m}{m^2 + m - 6} = \boxed{\frac{2m}{m+3}}$

$\frac{(m+2)(m-2)}{(m+3)(m+2)} \cdot \frac{2m(m+3)}{(m+3)(m-2)}$

(g) $\frac{x + 4}{2x + 10} - \frac{5}{x^2 - 25} = \boxed{\frac{x-6}{2(x-5)}}$

$\frac{(x-5)}{(x-5)} \cdot \frac{(x+4)}{2(x+5)} - \frac{5}{(x+5)(x-5)} \cdot \frac{2}{2}$

$= \frac{x^2 - x - 20 - 10}{2(x+5)(x-5)} = \frac{x^2 - x - 30}{2(x+5)(x-5)} = \frac{(x-6)(x+5)}{2(x+5)(x-5)}$

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2. Simplify the radical expression $3\sqrt{8} - 2\sqrt{18} + \sqrt{50}$ and provide a decimal approximation.

$= 3\cdot\sqrt{4}\cdot\sqrt{2} - 2\cdot\sqrt{9}\cdot\sqrt{2} + \sqrt{25}\cdot\sqrt{2}$

$= 3\cdot 2\sqrt{2} - 2\cdot 3\sqrt{2} + 5\sqrt{2} = 6\sqrt{2} - 6\sqrt{2} + 5\sqrt{2}$

$\boxed{5\sqrt{2}}$

≈ 7.07

3. Completely factor each polynomial below; Be sure your answer can not be factored further.

(a) $3x^3y^4 - 12x^2y^3 + 15x^4y$

$= 3x^2y(xy^3 - 4y^2 + 5x^2)$

$\boxed{3x^2y(xy^3 - 4y^2 + 5x^2)}$

(b) $16m^4 - 81$

$= (4m^2 - 9)(4m^2 + 9)$

$= (2m-3)(2m+3)(4m^2+9)$

$\boxed{(2m-3)(2m+3)(4m^2+9)}$

(c) $x^2 - 5x - 24$

$(x-8)(x+3)$

$\boxed{(x-8)(x+3)}$

	$2t$	3
$6t$	$12t^2$	$18t$
-1	$-2t$	-3

$-36t^2$

(d) $12t^2 + 16t - 3$

$\boxed{(2t+3)(6t-1)}$

(e) $x^3 - 5x^2 - 4x + 20$

$= x^2(x-5) - 4(x-5)$

$= (x-5)(x^2-4) = (x-5)(x+2)(x-2)$

$\boxed{(x-5)(x-2)(x+2)}$

	$2x$	-3
$2x$	$4x^2$	$-6x$
-3	$-6x$	9

$36x^2$

(f) $4x^2 - 12x + 9$

$(2x-3)(2x-3)$

$\boxed{(2x-3)^2}$

4. Solve each equation or inequality below. You must use an algebraic method; Guess and check earns no credit. If it's an inequality, graph the solution set on a number line. Give exact answers in simplified form (reduce fractions, simplify radicals, etc...).

(a) $5(x-3) + 2 = 5(2x-8) - 3$

$\Longrightarrow 5x - 15 + 2 = 10x - 40 - 3$

$\Longrightarrow 5x - 13 = 10x - 43$

$-10x + 13 \quad -10x + 13$

$\Longrightarrow$

$-5x = -30$

$\longrightarrow x = \frac{-30}{-5} = 6$

$\boxed{S = \{6\}}$

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(b) $\frac{2x}{x^2-4}+\frac{5}{x^2+2x}=\frac{2}{x-2}$

$(x+2)(x-2)\times\left[\frac{2x}{(x-2)(x+2)}+\frac{5}{x(x+2)}\right]=\left[\frac{2}{x-2}\right](x+2)(x-2)x$

$S=\{10\}$

$\longrightarrow \quad 2x^2+5(x-2)=2x(x+2)$

$\longrightarrow \quad 2x^2+5x-10=2x^2+4x$
$\qquad\qquad -4x\ +10 \qquad\quad -4x\ +10$

$\Longrightarrow \quad x=10$

(c) $\sqrt{2x+9}=x+5$

$S=\{-4\}$

CHECK

$\sqrt{2(-4)+9}\stackrel{?}{=}-4+5$

$\sqrt{-8+9}=1\ \checkmark$

$2x+9=(x+5)^2$

$\longrightarrow \quad 2x+9=x^2+10x+25$

$\longrightarrow \quad 0=x^2+8x+16$

$\longrightarrow \quad 0=(x+4)(x+4) \longrightarrow \quad x=-4$

(d) $(3x+2)^2=20$

$S=\left\{\frac{-2+2\sqrt{5}}{3},\ \frac{-2-2\sqrt{5}}{3}\right\}$

$\longrightarrow \quad 3x+2=\pm\sqrt{20}=\pm 2\sqrt{5}$

$\longrightarrow \quad 3x=-2\pm 2\sqrt{5} \quad \text{or} \quad x=\frac{-2\pm 2\sqrt{5}}{3}$

(e) $x(2x-3)=20$

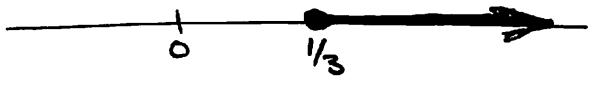
$S=\left\{-\frac{5}{2},\ 4\right\}$

$\longrightarrow \quad 2x^2-3x=20$

$\longrightarrow \quad 2x^2-3x-20=0$

$\longrightarrow \quad (2x+5)(x-4)=0 \longrightarrow \quad x=\frac{-5}{2} \text{ or } x=4$

(f) $5y\geq\frac{3y+4}{3}$



$\Longrightarrow \quad 15y\geq 3y+4$

$\longrightarrow \quad 12y\geq 4 \Longrightarrow y\geq\frac{4}{12} \text{ or } y\geq\frac{1}{3}$

(g) $-3<5-2x\leq 1$
$\quad -5\ \ -5 \qquad -5$

$\longrightarrow \quad -8<-2x\leq -4$

$\longrightarrow \quad \frac{-8}{-2}>\frac{-2x}{-2}\geq\frac{-4}{-2} \quad \text{or} \quad 4>x\geq 2 \quad \text{or} \quad 2\leq x<4$

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5. Solve the formula $z = \dfrac{2x - 3y}{4}$ for y.

$\rightarrow 4z = 2x - 3y$

$\rightarrow 4z - 2x = -3y$

$\rightarrow \dfrac{4z-2x}{-3} = y$

$$y = \frac{2x-4z}{3}$$

6. Given the quadratic equation $4m^2 + 8m + 1 = 0$, do the following:

(a) Solve the equation by the method of completing the square.

$$\frac{4m^2+8m}{4} = \frac{-1}{4}$$

$$m^2 + 2m = -\frac{1}{4}$$

$$m^2 + 2m + 1 = -\frac{1}{4} + 1 = \frac{3}{4}$$

$$(m+1)^2 = \frac{3}{4} \rightarrow m+1 = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2} \rightarrow m = -1 \pm \frac{\sqrt{3}}{2}$$

$$S = \left\{ -1 + \frac{\sqrt{3}}{2}, \; -1 - \frac{\sqrt{3}}{2} \right\}$$

(b) Solve the equation using the quadratic formula. Do not give calculator approximations; leave the answer in simplified radical form.

$a=4,\ b=8,\ c=1$

$b^2 - 4ac = 64 - 16 = 48$

$$m = \frac{-8 \pm \sqrt{48}}{8} = \frac{-8 \pm 4\sqrt{3}}{8} = \frac{\cancel{4}(-2 \pm \sqrt{3})}{\cancel{8}\,2}$$

$$S = \left\{ \frac{-2+\sqrt{3}}{2}, \; \frac{-2-\sqrt{3}}{2} \right\}$$

7. Answer the question by solving an equation or system of equations; To get credit you must declare a variable (or variables) and set up an equation (or system of equations) involving the variables and then solve the equation (or system of equations) to get the solutions to the problem.

Suppose Hector has 21 coins totaling \$3.45. If he has only dimes and quarters, how many of each type does he have?

LET d = # DIMES, q = # QUARTERS

① $d + q = 21 \rightarrow d = 21 - q$

② $10d + 25q = 345$

$\rightarrow 10(21-q) + 25q = 345$

$\rightarrow 210 - 10q + 25q = 345$

$\rightarrow 15q = 135 \rightarrow q = \dfrac{135}{15} = 9$ so $d = 21 - 9 = 12$

THERE ARE 9 QUARTERS AND 12 DIMES IN THE COLLECTION.

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8. Find an equation of the line that passes through the points $(3, 2)$ and $(-6, 5)$.

$m = \frac{\Delta y}{\Delta x} = \frac{5-2}{-6-3} = \frac{3}{-9} = -\frac{1}{3}$

$y - 2 = -\frac{1}{3}(x-3) \longrightarrow y - 2 = -\frac{1}{3}x + 1$

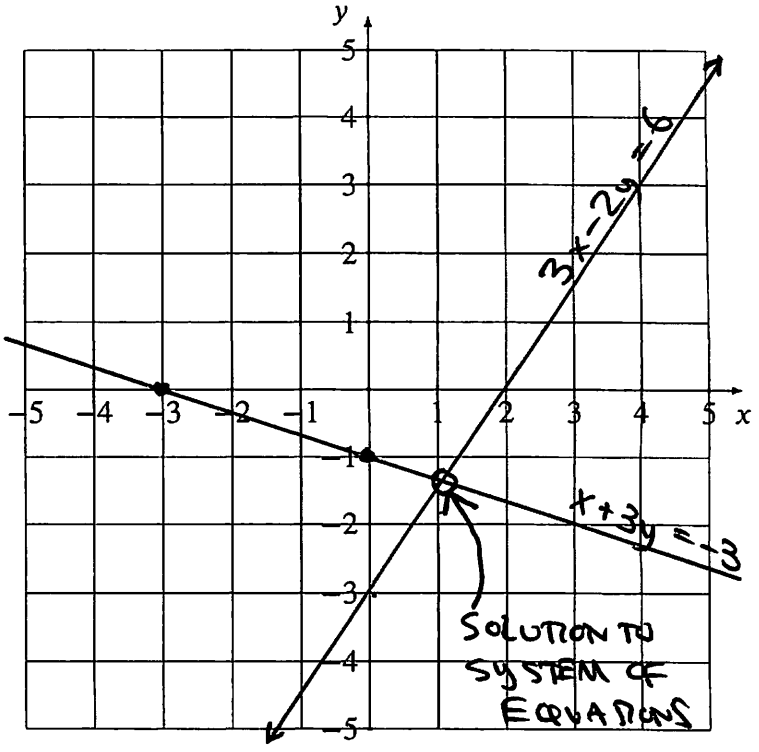
OR $y = -\frac{1}{3}x + 3$

$(y-2) = -\frac{1}{3}(x-3)$

OR

$y = -\frac{1}{3}x + 3$

9. The line graphed to the right has equation $3x - 2y = 6$.



2PT (a) Find the slope of this line.

$m = \frac{3}{2}$

3PT (b) Graph the line $x + 3y = -3$ as accurately as possible in the same coordinate system. Clearly indicate the solution to the system of the two graphed equations by circling it.

4PT (c) Find the exact solution, as reduced fractions, by solving the system of these two equations using either the method of substitution or elimination.

① $3x - 2y = 6$

② $x + 3y = -3 \longrightarrow x = -3 - 3y$ *

SUBSTITUTE * INTO ①

$3(-3-3y) - 2y = 6$

$\longrightarrow -9 - 9y - 2y = 6$

$\longrightarrow -11y = 15$

$\longrightarrow y = -\frac{15}{11}$

$x = -3 - 3\left(-\frac{15}{11}\right) = -3 + \frac{45}{11} = -\frac{33}{11} + \frac{45}{11} = \frac{12}{11}$

SOLUTION TO SYSTEM IS:

$\left(\frac{12}{11}, -\frac{15}{11}\right)$

$\frac{12}{12}$