



## What Makes an Equation Beautiful

By KENNETH CHANG

### Greatest Equations

Physics World magazine recently asked readers to send in nominations for the best equations of all time. Euler's equation was one of the most popular. It has wide application in understanding the motion of any type of wave, including light.

**Euler's equation** *Contains nine basic concepts of mathematics, elegantly.*

EXONENTS

THE SQUARE ROOT OF -1  
*Imaginary*

PI=3.14159...

BASE OF NATURAL LOGARITHMS  
=2.71828...

MULTIPLICATION

ADDITION

ONE

EQUALS

ZERO

*One respondent said of this equation:  
"What could be more mystical than an imaginary number  
interacting with real numbers to produce nothing?"*

CONSIDER a verbal description of the effect of gravity: drop a ball, and it will fall.

That is a true enough fact, but fuzzy in the way that frustrates scientists. How fast does the ball fall? Does it fall at constant rate, or accelerate? Would a heavier ball fall faster? More words, more sentences could provide details, swelling into an unwieldy yet still incomplete paragraph.

The wonder of mathematics is that it captures precisely in a few symbols what can only be described clumsily with many words. Those symbols, strung together in meaningful order,

# The Other Winners

Euler's equation and Maxwell's received the most votes. The remaining are listed in the order of number of votes received.

Maxwell's equations <i>Taken together</i>	$\nabla \cdot \mathbf{D} = \rho$ $\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$
Newton's Second Law	$\mathbf{F} = m\mathbf{a}$
Pythagorean theorem	$a^2 + b^2 = c^2$
Schrödinger equation	$H\Psi = E\Psi$
Einstein's equation	$E = mc^2$
Boltzmann equation	$S = k \ln W$
One plus one	$1+1=2$
Principle of least action	$\delta S = 0$
DeBroglie's equation	$p = \frac{h}{\lambda}$
Fourier transform	$f(x) = \int_{-\infty}^{\infty} F(k) e^{2\pi i k x} dk$
Einstein's general theory of relativity	$G_{\mu\nu} = 8\pi G T_{\mu\nu}$
Circumference of a circle	$C = 2\pi r$
Dirac equation	$i\gamma \cdot \partial\Psi = m\Psi$
Riemann zeta function	$\zeta(s) = \prod_{(p)} \left[ \frac{p^s}{p^s - 1} \right]$
Hubble's Law	$v = H_0 d$

make equations - which in turn constitute the world's most concise and reliable body of knowledge. And so it is that physics offers a very simple equation for calculating the speed of a falling ball.

Readers of Physics World magazine recently were asked an interesting question: Which equations are the greatest?

Dr. Robert P. Crease, a professor of philosophy at the State University of New York at Stony Brook and a historian at Brookhaven National Laboratory, posed the question in his Critical Point column and received 120 responses, nominating 50 different equations. Some were nominated for the sheer beauty of their simplicity, some for the breadth of knowledge they capture, others for historical importance. In general, Dr. Crease said, a great equation "reshapes perception of the universe."

The mathematical equation providing the speed of a falling ball is just four symbols long:  $v = gt$ .

With it, you can calculate the ball's speed 2.5 seconds after release. (That's  $g$ , the acceleration of gravity, which is 32 feet per second squared, multiplied by 2.5 seconds, giving an answer of 80 feet per second.)

This equation, a mainstay of high school physics, was not among those nominated as the greatest of all time, which is not surprising, because its use is limited.

The pull of gravity varies with distance from the Earth's surface, and the equation also

Simplest ratio	$\frac{a}{b} = \frac{c}{d}$
Ideal gas law	$PV = nRT$
Balmer series	$\frac{1}{\lambda_n} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right)$
Planck equation	$E = hv$

suggests that an object's speed could go on increasing toward infinity, past the known limit of the speed of light.

The top vote-getters in the magazine poll were Maxwell's equations - a set of four that describe the interplay between electric and magnetic fields - and Euler's equation, a purely mathematical construct that finds

wide use in theoretical physics.

"It combines rational and irrational numbers to get zero," Dr. Crease said. "It's bizarre."

Among the other nominees were the all-familiar  $E=mc^2$  from Einstein, which equates energy and matter; the Pythagorean theorem; and Isaac Newton's  $F=ma$ .

Prominent scientists have their own favorites. Dr. Brian Greene, a theorist at Columbia University and author of "The Elegant Universe," cites Einstein's general relativity equations, which describe how matter warps the fabric of space, and the Schrödinger equation, the fundamental equation of quantum mechanics.

"With a mere handful of symbols, those equations describe almost all phenomena in the universe," he said. "It is so amazing how so much of the universe is encapsulated in a few symbols."

Dr. Neil deGrasse Tyson, director of the Hayden Planetarium, said he was disappointed that  $E=mc^2$  did not receive more votes. "I think the general physics community, they're a little bored with the equation," he said. "It's risen to the level of icon that people no longer pay attention to."

But Dr. Tyson said that the equation was a fundamental underpinning not only of the universe, but also of the first five chapters of his book "Origins."

"It's simple, yet profound," he said. "I'd be less impressed if it were a big complicated equation."

A half-dozen of Dr. Crease's respondents, including Richard Harrison of Calgary, Alberta, chose one of the simplest possible equations.

Mr. Harrison wrote: " '1 + 1 = 2' is the fairy tale of mathematics, the first equation I taught

my son, the first expression of the miraculous power of the mind to change the real world. I remember my son holding up the index finger, the 'one finger,' of each hand as he learned the expression, and the moment of wonder, perhaps his first of true philosophical wonder, when he saw that the two fingers, separated by his whole body, could be joined in a single concept in his mind."