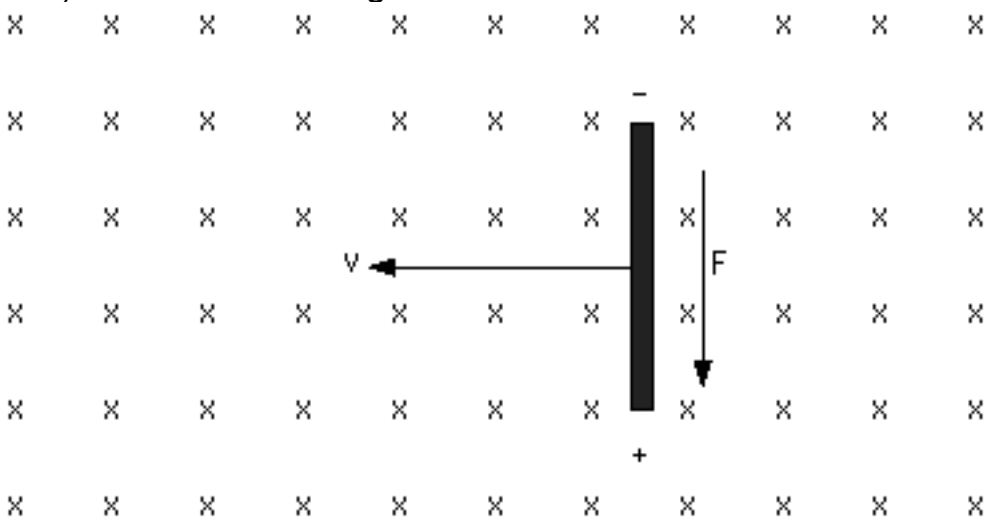


Electromagnetic Induction

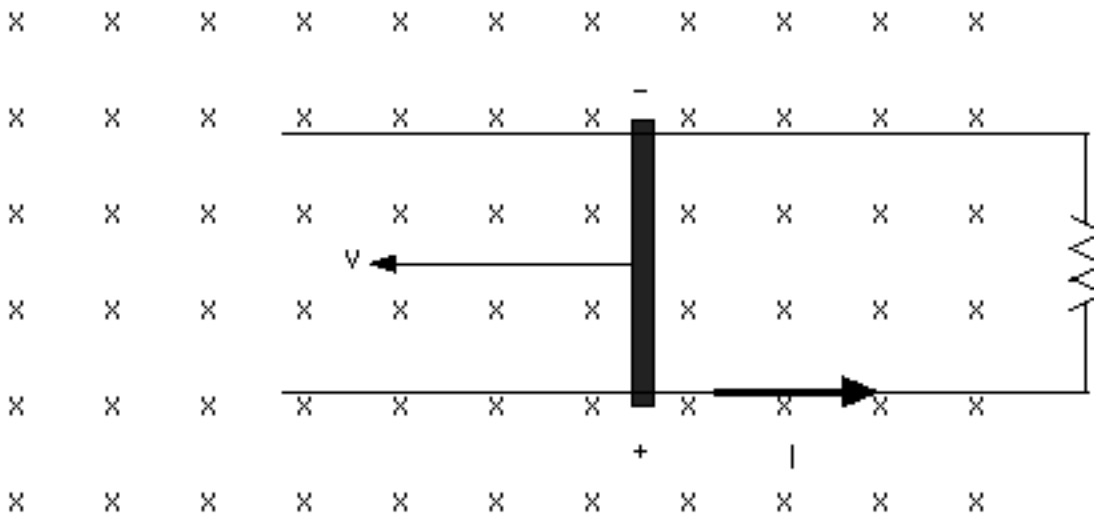
What generates an electric field? A charge, of course. But as we will see today (and have already seen in 220) an electric field is also generated by a magnetic field; whenever magnetic field lines move or change in any way they will generate an electric field, an **induced electric field**. This electric field exerts electric forces on charges. However it differs from an electrostatic field because it is not conservative - that is if you integrate E over a closed path (which is simply giving you the change in the voltage) is not zero for an induced electric field, rather the path intergral for the induced electric field is the **induced emf**.

Let's start by considering the emf produced by the motion of a path (in this case a rod) in a constant magnetic field



a positive charge q will feel a magnetic force qvB on in a downward direction, so positive charges will flow to the bottom (and negative ones, the electrons) will flow to the top. This can't go on forever because eventually the electrostatic force (upwards for the positive charge) will equal the magnetic force and the flow will stop.

However, if the ends of the rod are in sliding contact with a pair of wires that provide a stationary return path, then the electrons will not accumulate at the top but flow continuously around the circuit - the moving rod acts as a pump of electricity, that is as a source of emf. In this case the upper end of the rod is the negative terminal, the lower end the positive



The emf associated with the rod is the work done by the driving force on a unit positive charge that passes from the negative end of the rod to the positive

the driving force per unit charge is $qvB/q = vB$

the length of the rod is l

the induced emf, E , is

$E = vBl$ (we write this as Blv in 220)

Also note that since charges are flowing in the rod and there is a magnetic field, there will be a force on these charges. This force will be to the right and it is cancelled by the constraining forces that the sides of the rod exert on the charges preventing their escape (from the sides, they do escape from the ends). If it were not for this cancellation of part of the magnetic force, the work done by the magnetic force would be zero.

The energy delivered to the current by the emf does not ultimately come from the magnetic field but rather from whatever device is responsible for moving the rod in the magnetic field (a push or pull of some type). The magnetic field does provide the role of 'middle man' in transferring the energy from mechanical to electric. Also note that if we did not provide a continued push on the rod, it would slow down as its kinetic energy was converted into electric energy.

The quantity Blv was given an interesting interpretation by Faraday

$lv =$ the area the rod sweeps across per unit time

B - number of magnetic field lines per unit area

Bv = number of magnetic field lines the rod sweeps (or cuts) across per unit time

so our equation can be understood as

E = the number of magnetic field lines cut per second
= rate of cutting magnetic field lines

So we should get an induced emf whenever we cut the field lines and this will let us generalize Faraday's Law in a bit. But first another example of a motional emf

If we look at the top of the rod, we can see it will be negative. But this is also true for the bottom of the rod, so where is the positive - it's in the middle

a homopolar generator

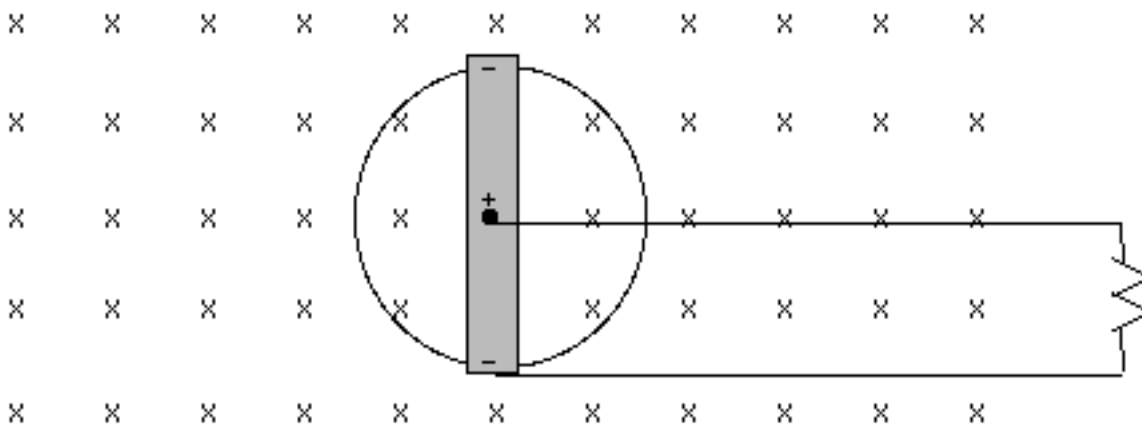
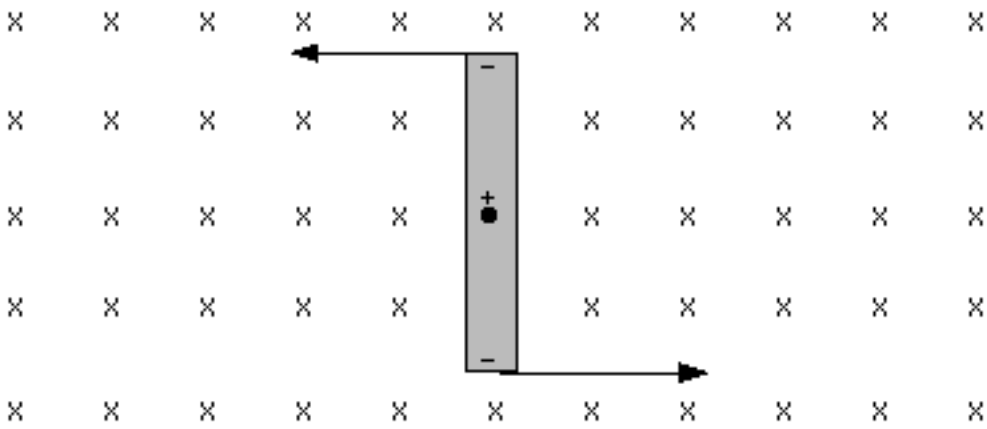
HW Question - if the rod has length $2L$ and is rotating thru a magnetic field B with an angular velocity ω , what is the induced emf between the center of the rod and one end? Between the two ends?

Real homopolar generators use rotating disks rather than rods. This does not effect the emf but cuts way down on the internal resistance. They are used in situations where large currents at small voltages are needed, such as electroplating.

Another interesting motional emf is the magnetohydrodynamic generator (MHD). This is a rectangular channel thru which flows a very hot, ionized gas. The channel is then placed in a magnetic field. The gas being very hot is moving extremely fast, say 1000 m/s and the magnetic field is say 2 T and the channel is say 0.5 m wide

$$E = Bv = 2 \cdot 1000 \cdot .5 = 1000 \text{ V}$$

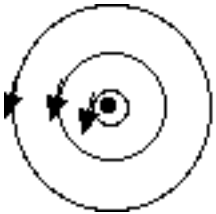
MHD generators are coming on line as auxiliary generators to fossil fuel power plants.



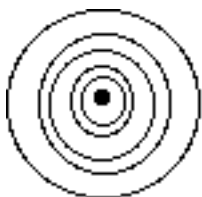
Faraday's Law

We have just seen that if we move a rod thru a magnetic field an emf will be induced. It should make no difference if the rod were stationary and the magnetic field moved, as long as there is relative motion between the rod and the magnet.

But there is another way we can cut magnetic field lines - we hold the rod constant and increase or decrease the magnetic field by increasing or decreasing the current in an electromagnet



magnetic field lines
created by long straight
wire with a little current



magnetic field for a stronger current

Lets look at the magnetic field of a long straight current carrying wire - they circle the wire. If we increase the magnetic field we naturally get more field lines. How do we increase the magnetic field - we increase the current in the wire. As we increase the current, it makes new, small circles of field lines in the immediate vicinity of the wire and the older ones expand and move outwards, just like the ripples in a pond - notice that the pattern grows from the inside out [If we were to decrease the current, the field lines must decrease but it cannot be that the outer ones start to move back; the change must come from the closest to the current. what happens is that as we decrease the current it makes new small circles of field lines in the opposite direction and these move outwards, gradually cancelling the original field.

What is important here is that the field lines are moving - so if we put a rod in the vicinity of the current carrying wire, the moving field lines will sweep across it and be cut by it (like producing a piece of bread by moving the loaf of bread across the knife). This allows us to generalize Faraday's Law

The induced emf along any moving or fixed path in a constant of a changing magnetic field equals the rate at which magnetic field lines cut across the path

E = rate of cutting magnetic field lines

Note that we said path, not rod or wire. The emf will exist between the ends of the path whether the path lies in a rod or in empty space - whenever a unit charge moves along this path it will gain E amount of energy from the induced electric field. Of course in a real case we would want the wire or rod.

For a closed path, Faraday's law is stated in terms of the magnetic flux. The magnetic flux thru any given area is simply the number of field lines intercepted by the area and is generally written as an integral

$$\Phi_B = \int B \cdot dS$$

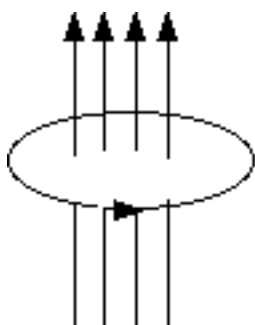
The unit of magnetic flux we use in 220 is the Tm^2 , which is actually a weber, Wb

We have to take account of whether the field line is entering the area bounded by the path or leaving it

The net rate of cutting the field lines is equal to the rate of change of the number of field lines intercepted by the area bounded by the path, that is it is equal to the rate of the change in the magnetic flux. So Faraday's law can be written

$$E = - \frac{d\Phi_B}{dt}$$

Why the minus - the sign of the induced emf is related to the sign of the flux and the sign of the change in the flux. The sign of the magnetic flux is given by yet another right hand rule - wrap your fingers around the path in the direction in which we are determining the emf - the flux is positive if the magnetic field points in the direction of your thumb, negative otherwise



A closed path in a changing magnetic field
The magnetic field must point in the direction shown to have a positive flux

Example - A rectangular coil of 150 loops of wire forming a closed circuit measures 0.2 m by 0.1 m and has a resistance of 5 ohms. The coil is placed in a perpendicular downwards magnetic field of an electro magnetic which is suddenly switched off. If the magnetic field changes by 20 T per second, what is the induced emf, the induced current and the direction of the current?

$$E = - d\Phi/dt$$

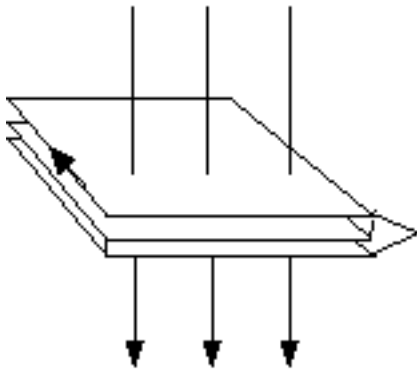
the $d\Phi = d(BA \cos \theta)$ $\theta = 0$, $\cos \theta = 1$

$A = \text{area enclosed by loop} = N l w$

$$E = - d(BNlw)/dt = - Nlw dB/dt = 150 \times .2 \times .1 \times 20 = 60 \text{ V (magnitude)}$$

$$I = E/R = 60/5 = 12 \text{ A}$$

Now for the direction of I (and of E)

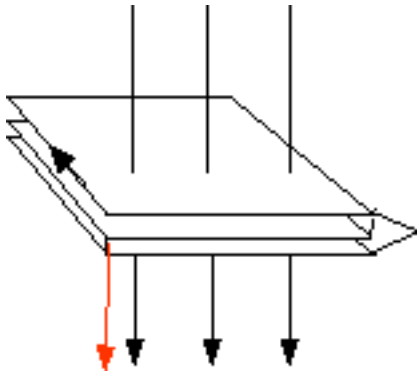


To determine the direction of the induced current, let us say the emf is in the direction shown by the arrow on the loop. By the right hand rule the field lines shown make a positive contribution to the flux. Since the magnetic field is decreasing, the change in the flux will be negative and therefore the emf will be positive.

There is a simpler way of finding the direction of the emf, Lenz's law

The induced emf (and induced current) are always in such a direction so as to oppose the change that produced it.

From our example, the change in was a decrease in B down. The induced emf/current will want to lessen this decrease, which it will do by making a magnetic field down



Of course, if Lenz's law were not true, energy would not be conserved.