

Solutions to Study Guide for Final Exam

This is how you will see these types of problems on the actual test

1. Solve the system by substitution.

$$\begin{aligned} 6x - 5y &= -8 \\ x + 3y &= 14 \end{aligned}$$

a) Circle the correct description of the system.

- i. Dependent system
- ii. One solution system
- iii. Inconsistent system.

b) What is the solution of the system of the system of equations? Circle the correct choice below, and if necessary, fill in the answer box to complete your choice.

- i. (2, 4) (Write the ordered pair.)
- ii. There are infinitely many solutions.
- iii. There is no solution.

$$\begin{array}{r} x + 3y = 14 \\ -3y \quad -3y \\ \hline x = -3y + 14 \end{array}$$

$$\begin{aligned} 6(-3y + 14) - 5y &= -8 \\ -18y + 84 - 5y &= -8 \\ -23y + 84 &= -8 \\ -23y &= -92 \end{aligned}$$

$$\begin{array}{r} -23y = -92 \\ \hline -23 \quad -23 \\ \hline y = 4 \end{array}$$

$$y = 4$$

(2, 4)

$$\begin{array}{r} x + 3y = 14 \\ x + 3(4) = 14 \\ x + 12 = 14 \\ -12 \quad -12 \\ \hline x = 2 \end{array}$$

2. Solve the system by elimination.

$$\begin{aligned} 6x - 9y &= -3 \\ -10x + 15y &= 5 \end{aligned}$$

a) Circle the correct description of the system.

- iv. Dependent system
- v. One solution system
- vi. Inconsistent system.

b) What is the solution of the system of the system of equations? Circle the correct choice below, and if necessary, fill in the answer box to complete your choice.

- iv. _____ (Write the ordered pair.)
- v. There are infinitely many solutions.
- vi. There is no solution.

$$\begin{array}{r} 5(6x - 9y) = (-3)5 \\ 3(-10x + 15y) = (5)3 \\ \hline 30x - 45y = -15 \\ -30x + 45y = 15 \\ \hline 0 = 0 \text{ true statement} \end{array}$$

0 = 0 true statement

3. Factor. If the polynomial is prime, say so. Show all work for full credit. Circle your final answers.

<p>a) $x^2 - 11x + 18$</p> <p>$(x-9)(x-2)$</p>	<p>b) $2x^2 + 20x + 42$</p> <p>$2(x^2 + 10x + 21)$</p> <p>$2(x+7)(x+3)$</p>									
<p>c) $25x^2 - 16$</p> <p>$(5x-4)(5x+4)$</p>	<p>d) $x^2 - 4xy - 21y^2$</p> <p>$(x-7y)(x+3y)$</p>									
<p>e) $5x^5 + 45x^4 + 70x^3$</p> <p>$5x^3(x^2 + 9x + 14)$</p> <p>$5x^3(x+7)(x+2)$</p>	<p>f) $3xy^2 - 48x$</p> <p>$3x(y^2 - 16)$</p> <p>$3x(y+4)(y-4)$</p>									
<p>g) $-x^2 - 4x - 3$</p> <p>$-(x^2 + 4x + 3)$</p> <p>$-(x+3)(x+1)$</p>	<p>h) $3x^3 + x^2 + 27x + 9$</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td style="text-align: center;">$3x$</td> <td style="text-align: center;">1</td> </tr> <tr> <td style="text-align: center;">x^2</td> <td style="text-align: center;">$3x^3$</td> <td style="text-align: center;">x^2</td> </tr> <tr> <td style="text-align: center;">9</td> <td style="text-align: center;">$27x$</td> <td style="text-align: center;">9</td> </tr> </table> <p>$(x^2 + 9)(3x + 1)$</p> <p>note: $x^2 + 9$ cannot be factored further</p>		$3x$	1	x^2	$3x^3$	x^2	9	$27x$	9
	$3x$	1								
x^2	$3x^3$	x^2								
9	$27x$	9								

be factored further

4. Solve. Write your answers as integers or simplified fractions. Show all work & circle your answers. LCD = (w-5)(w+2)

a) $\frac{5}{x} + \frac{3}{x-2} = \frac{7}{x} \cdot (x-2)$ LCD = $x(x-2)$

$$5(x-2) + 3x = 7(x-2)$$

$$5x - 10 + 3x = 7x - 14$$

$$8x - 10 = 7x - 14$$

$$\begin{array}{r} 8x - 10 = 7x - 14 \\ +10 \quad +10 \\ \hline 8x = 7x - 4 \end{array}$$

$$\begin{array}{r} 8x = 7x - 4 \\ -7x \quad -7x \\ \hline x = -4 \end{array}$$

$x = -4$

$x = -4$

excluded values

$w = -2$ and $w = 5$

$\frac{w}{w+2} + \frac{7}{w-5} = \frac{14}{w^2-3w-10} = \frac{14}{(w-5)(w+2)}$

$$w(w-5) + 7(w+2) = 14$$

$$w^2 - 5w + 7w + 14 = 14$$

$$\begin{array}{r} w^2 + 2w + 14 = 14 \\ -14 \quad -14 \\ \hline w^2 + 2w = 0 \end{array}$$

$$w(w+2) = 0$$

$$w = 0 \text{ or } w = -2$$

$$w = 0 \text{ or } w = -2$$

$$w = 0 \text{ or } w = -2$$

but $w = -2$ is an excluded value!

only soln:
 $w = 0$

c) $-x^2 - x + 3 = -9$
 $+9 \quad +9$

$$-1(-x^2 - x + 12) = 0$$

$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

$$x+4 = 0 \text{ or } x-3 = 0$$

$$\begin{array}{r} x+4 = 0 \\ -4 \quad -4 \\ \hline x = -4 \end{array}$$

$$\begin{array}{r} x-3 = 0 \\ +3 \quad +3 \\ \hline x = 3 \end{array}$$

$x = -4$ or $x = 3$

d) $(x+1)(x-2) = 4$

$$x^2 - 2x + x - 2 = 4$$

$$\begin{array}{r} x^2 - x - 2 = 4 \\ -4 \quad -4 \\ \hline x^2 - x - 6 = 0 \end{array}$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x-3 = 0 \text{ or } x+2 = 0$$

$$\begin{array}{r} x-3 = 0 \\ +3 \quad +3 \\ \hline x = 3 \end{array}$$

$$\begin{array}{r} x+2 = 0 \\ -2 \quad -2 \\ \hline x = -2 \end{array}$$

$x = 3$ or $x = -2$

e) $-\frac{1}{2}x^2 + \frac{7}{2}x + 12 = 3$ LCD = -2

$$\begin{array}{r} x^2 - 7x - 24 = -6 \\ +6 \quad +6 \\ \hline x^2 - 7x + 18 = 0 \end{array}$$

$$(x-9)(x+2) = 0$$

$$x-9 = 0 \text{ or } x+2 = 0$$

$$\begin{array}{r} x-9 = 0 \\ +9 \quad +9 \\ \hline x = 9 \end{array}$$

$$\begin{array}{r} x+2 = 0 \\ -2 \quad -2 \\ \hline x = -2 \end{array}$$

$x = 9$ or $x = -2$

because I want to make the leading coefficient positive

f) $2x^3 - 3x^2 - 50x + 75 = 0$

$2x^3$	$-3x^2$
$-50x$	75

$$(x^2-25)(2x-3) = 0$$

$$(x-5)(x+5)(2x-3) = 0$$

$$x-5 = 0 \text{ or } x+5 = 0 \text{ or } 2x-3 = 0$$

$$\begin{array}{r} x-5 = 0 \\ +5 \quad +5 \\ \hline x = 5 \end{array}$$

$$\begin{array}{r} x+5 = 0 \\ -5 \quad -5 \\ \hline x = -5 \end{array}$$

$$\begin{array}{r} 2x-3 = 0 \\ +3 \quad +3 \\ \hline 2x = 3 \\ \hline x = \frac{3}{2} \end{array}$$

$x = 5$ or $x = -5$ or $x = \frac{3}{2}$

5. Perform the indicated operations. Write your answers using integers or simplified fractions. Show all work and simplify your answers. You may leave your answers in factored form, as appropriate.

a) $(4p + 8q) + (4p - 9q)$

$$\boxed{8p - q}$$

b) $(3t - 5w)^2$

$$(3t - 5w)(3t - 5w)$$

$$9t^2 - 15tw - 15tw + 25w^2$$

$$\boxed{9t^2 - 30tw + 25w^2}$$

c) $\frac{5}{x} + \frac{3}{x-2} - \frac{7}{x}$ ← not an equation - need to find equiv. fractions w/ common denominator
 LCD = $x(x-2)$

$$\frac{5(x-2) + 3x - 7(x-2)}{x(x-2)}$$

$$\frac{5x - 10 + 3x - 7x + 14}{x(x-2)}$$

$$= \boxed{\frac{x+4}{x(x-2)}}$$

d) $-5xy(3x^2 - 7xy + 9y^2)$

$$\boxed{-15x^3y + 35x^2y^2 - 45xy^3}$$

e) $2(x+3)^2 - 4$ multiply out!

$$2(x^2 + 6x + 9) - 4$$

$$2x^2 + 12x + 18 - 4$$

$$\boxed{2x^2 + 12x + 14}$$

f) $5p^3t(-6p^3t)$

$$\boxed{-30p^6t^2}$$

g) $(11x - 7) + (5x + 8)$

$$\boxed{6x - 15}$$

h) $(5x - 4y)(3x - 6y)$

$$15x^2 - 30xy - 12xy + 24y^2$$

$$\boxed{15x^2 - 42xy + 24y^2}$$

i) $\frac{-6x+36}{x^2+7x+12} \cdot \frac{x^2-16}{-3x+18}$ ← should be 18

$$2 \cdot \frac{-6(x-6)}{(x+3)(x+4)} \cdot \frac{(x-4)(x+4)}{-3(x-6)}$$

$$\boxed{\frac{2(x-4)}{(x+3)}}$$

j) $\frac{x^2-64}{x^2-9x+20} \div \frac{x^2-15x+56}{x^2-4x-5}$

$$\frac{x^2-64}{x^2-9x+20} \cdot \frac{x^2-4x-5}{x^2-15x+56}$$

$$\frac{(x-8)(x+8)}{(x-4)(x-5)} \cdot \frac{(x-5)(x+1)}{(x-7)(x-8)}$$

$$\boxed{\frac{(x+8)(x+1)}{(x-4)(x-7)}}$$

6. Simplify. Use integers or simplified fractions in your answers. Show all work & circle your answers.

<p>a) $\frac{48x^6y^4}{8x^5y^{-3}}$</p> <p>$6xy^7$</p>	<p>b) $\frac{\frac{3}{x^2-16}}{\frac{4}{x+4}}$</p> <p>$\frac{3}{x^2-16} \cdot \frac{(x+4)}{4}$</p> <p>$\frac{3}{(x-4)(x+4)} \cdot \frac{(x+4)}{4}$</p> <p>$\frac{3}{4(x-4)}$</p>	<p>c) $(4x^{-2}y)^3$</p> <p>$64x^{-6}y^3$</p> <p>$\frac{64y^3}{x^6}$</p>
<p>d) $-5c^4(c^2)^5$</p> <p>$-5c^4c^{10}$</p> <p>$-5c^{14}$</p>	<p>e) $\frac{5-\frac{3}{x}}{4-\frac{1}{x}} \cdot \frac{x}{x}$ $LCD=x$</p> <p>$\frac{5x-3}{4x-1}$</p>	<p>f) $\frac{(2a^{-6}b)^{-3}}{(3cd^{-2})^2}$</p> <p>$\frac{1}{(2a^{-6}b)^3(3cd^{-2})^2}$</p> <p>$\frac{1}{8a^{-18}b^39c^2d^{-4}}$</p> <p>$\frac{a^{18}d^4}{72b^3c^2}$</p>

7. A batter hits a baseball ball into the air. The height h (in feet) of the baseball after t seconds is given by $h = -16t^2 + 80t + 4$.

a) Predict when the baseball is at a height of 68 feet. Show all work and write your answer in a complete sentence in the context of the problem.

$68 = -16t^2 + 80t + 4$

-68

$-16t^2 + 80t - 64 = 0$

-16

$t^2 - 5t + 4 = 0$

$h = 68$

$(t-4)(t-1) = 0$

$t-4 = 0$ or $t-1 = 0$

$+4 \quad +4$ $+1 \quad +1$

$t = 4$ $t = 1$

The baseball is at a height of 68 ft at 1 + 4 seconds after it's hit.

b) How high is the baseball after 2 seconds? Show all work and write your answer in a complete sentence in the context of the problem.

$t = 2$

$h = -16(2)^2 + 80(2) + 4$

$h = -16(4) + 160 + 4$

$h = -64 + 164$

$h = 100$

After 2 seconds, the baseball is at a height of 100 feet.

8. In 2003, Americans consumed an average of 16.3 pounds of fish and shellfish per year. What is this average in ounces per day?

Equivalent Units

Length

- 1 inch = 2.54 centimeters
- 1 foot = 12 inches
- 1 yard = 3 feet
- 1 mile = 5280 feet
- 1 mile = 1.61 kilometers

Volume

- 1 cup = 8 ounces
- 1 quart = 4 cups
- 1 quart = 0.946 liter
- 1 gallon = 4 quarts

Weight

- 1 gram = 1000 milligrams
- 1 pound = 16 ounces

Time

- 1 year = 365 days

- a) Translate and solve.
 b) Write your answer in a complete sentence.

$$\frac{16.3 \text{ pounds}}{\text{year}} \cdot \frac{1 \text{ yr}}{365 \text{ days}} \cdot \frac{16 \text{ oz}}{1 \text{ pound}}$$

$$= \frac{16.3 \cdot 16 \text{ oz}}{365 \text{ day}} = .7145 \text{ oz/day}$$

b) 16.3 pounds of fish & shellfish per year is equivalent to .7145 ounces per day.

9. The weight of an object on Planet A and the weight of the same object on the Planet B are proportional. An astronaut who weighs 180 pounds on Planet A weighs 22.5 pounds on the Planet B. What is the weight of a person on Planet A if they weigh 28.9 pounds on the Planet B? Round your answer to the nearest integer as needed.

Planet A
 Planet B

$$\frac{180 \text{ (Planet A)}}{22.5 \text{ (Planet B)}} = \frac{x}{28.9}$$

Proportional implies these ratios are equivalent

Cross multiply

$180 \cdot 28.9 = 22.5x$

$$\frac{5202}{22.5} = \frac{22.5x}{22.5}$$

$x = 231.2 \rightarrow 231 \text{ rounded}$

A person weighs 231 pounds on planet A if they weigh 28.9 pounds on planet B

10. The numbers of men and women who earned a bachelor's degree are listed in the table below for various years. Let n be the number of people (in thousands) who earned a bachelor's degree in the year that is t years since 1980.

Year	Number of People Who Earned a Bachelor's Degree (thousands)	
	Women	Men
1980	456	474
1985	497	483
1990	560	492
1995	634	526
2000	708	530
2002	742	550

Reasonable models for the women and men are

$n = 13.28t + 440.09$	Women
$n = 3.42t + 468.14$	Men

Use substitution or elimination to estimate when the number of women who earned a bachelor's degree was equal to the number of men who earned a bachelor's degree. What was the number of people?

Remember to show all work and answer the question in a complete sentence for full credit.
 Round your intermediate answers to two decimal places and final answers to the nearest counting numbers.

$$\begin{array}{r} 13.28t + 440.09 = 3.42t + 468.14 \\ -3.42t \qquad \qquad -3.42t \\ \hline 9.86t + 440.09 = 468.14 \\ -440.09 \quad -440.09 \\ \hline 9.86t = 28.05 \\ \frac{9.86t}{9.86} = \frac{28.05}{9.86} \\ t = 2.84 \end{array}$$

$$\begin{array}{r} 9.86t + 440.09 = 468.14 \\ -440.09 \quad -440.09 \\ \hline 9.86t = 28.05 \\ \frac{9.86t}{9.86} = \frac{28.05}{9.86} \\ t = 2.84 \end{array}$$

$$\frac{9.86t}{9.86} = \frac{28.05}{9.86}$$

$$t = 2.84$$

$$n = 13.28(2.84) + 440.09$$

$$n = 477.85$$

Soln (2.84, 477.85)
 which rounds to (3, 478)

$t = 3$ means 1984
 In 1984, the number of men & women who earned bachelors degrees was the same. That year, they each earned 478,000 bach. degrees.

11. In addition, there will be a problem that you have never seen before. Use what you have learned in this class in addition to your problem solving skills to solve it and explain.