

1. Prove the following trig identities.

$$\text{a) } \sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$\text{b) } \frac{\sin t}{1 + \cos t} = \frac{1 - \cos t}{\sin t}$$

$$\text{c) } \cos 2x = \frac{2 - \sec^2 x}{\sec^2 x}$$

$$\text{d) } \frac{\sin(x+y)}{\cos(x-y)} = \frac{\cot x + \cot y}{1 + \cot x \cot y}$$

2. Use the addition formula $\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$ to derive the following identity for the average rate of change of the tangent function:

$$\frac{\tan(x + h) - \tan x}{h} = \frac{1}{\cos^2 x} \left(\frac{\sin h}{h} \right) \frac{1}{\cos h - \sin h \tan x}$$

HOMEWORK

Prove the following trig identities.

$$\text{a) } \csc \theta \cos^2 \theta + \sin \theta = \csc \theta$$

$$\text{b) } \frac{\sin x + \tan x}{1 + \cos x} = \tan x$$

$$\text{c) } \frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} = 2 \csc x$$

$$\text{d) } 2 \cos^3 x - \cos x = \frac{\cos^2 x - \sin^2 x}{\sec x}$$

$$\text{e) } \frac{\tan^2 t + 1}{\tan t \csc^2 t} = \tan t$$